

More hidden heavy quarkonium molecules and their discovery decay modes

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To validate the molecular description of the observed $Z_b(10610)/Z_b(10650)$ and $Z_c(3900)/Z_c(4025)$, it is valuable to investigate their counterparts, denoted as $Z_{QV}^{(\prime)}$ in this work, and the corresponding decay modes. In this work, we present an analysis of the $Z_{QV}^{(\prime)}$ using flavor symmetry. We also use the effective Lagrangian based on the heavy quark symmetry to explore the rescattering mechanism and calculate the partial widths for the isospin conserved channels $Z_{QV}^{(\prime)} \rightarrow \eta_Q V$. The predicted partial widths are of an order of MeV for $Z_{QV} \rightarrow \eta_Q V$, which correspond to branching ratios of the order of $10^{-2} \sim 10^{-1}$. For $Z_{QV}' \rightarrow \eta_Q V$, the partial widths are a few hundreds of keV and the branching ratios are about 10^{-3} . Future experimental measurements can test our predictions on the partial widths and thus examine the molecule description of heavy quarkoniumlike exotic states.

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I. INTRODUCTION

In the past few years, experiments have made great progress on the observations of XYZ states and some of them cannot be accommodated in the quark model as $Q\bar{Q}$ mesons [1]. Among these states, charged charmoniumlike and bottomoniumlike have attracted special attention due to their four-quark nature [2–5]. In 2011, the Belle Collaboration reported two charged bottomoniumlike structures, $Z_b^\pm(10610)$ and $Z_b^\pm(10650)$, in the $\Upsilon(nS)\pi^\pm$ ($n = 1, 2, 3$) and $h_b(mP)\pi^\pm$ ($m = 1, 2$) invariant mass spectra of $e^+e^- \rightarrow \Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$ and $e^+e^- \rightarrow \Upsilon(5S) \rightarrow h_b(mP)\pi^+\pi^-$ [6, 7]. The measured masses of $Z_b^\pm(10610)$ and $Z_b^\pm(10650)$ are slightly above the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds, respectively. In 2013, the BESIII Collaboration observed a new charged state $Z_c^\pm(3900)$ in the $J/\psi\pi^\pm$ invariant mass spectrum of $Y(4260) \rightarrow J/\psi\pi^+\pi^-$ [8]. Later, this new charged charmoniumlike structure was also observed in the $J/\psi\pi^\pm$ invariant mass spectrum by the Belle Collabo-

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ration [9] and confirmed by an analysis based on the CLEO data at the energy of 4.17 GeV [10]. Another new charged structure, $Z_c^\pm(4025)$, was reported in the process $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$ at $\sqrt{s} = 4.26$ GeV by the BESIII Collaboration [11]. Different from the other charmoniumlike and bottomoniumlike states, such as $X(3872)$, $Y(4260)$, etc., $Z_c^{(\prime)}$ and $Z_b^{(\prime)}$ are electric charged states and thus cannot be heavy quarkonium states. A charged combination could be formed by a state composed of four quarks, so these states may be ideal candidates for exotic hadrons beyond the conventional $Q\bar{Q}$ mesons.

The discoveries of these charged $Z_c^{(\prime)}$ and $Z_b^{(\prime)}$ states immediately initiated numerous studies of their structure, production, and decay mechanisms. Since the masses of the discovered states lie slightly above the meson-meson thresholds, it has been suggested that they are S -wave molecular states of heavy meson pair thresholds, i.e., $B^{(*)}\bar{B}^*$ and $D^{(*)}\bar{D}^*$ [12–16]. Besides this explanation, these states also have been identified as tetraquark states based on the fact that these particles have a typical hadronic total width of a few tens of MeV [17–20]. Besides the spectrum study, the production and decay of $Z_c^{(\prime)}$ and $Z_b^{(\prime)}$ states have also been investigated extensively [21–30].

On one hand, the molecular description can explain the existing data on $Z_c^{(\prime)}/Z_b^{(\prime)}$; on the other hand, this interpretation has predicted more counterpart states to be discovered. In this work, we will investigate these new hidden heavy quarkonium states and more particularly the isospin conserved processes $Z_{QV}^{(\prime)} \rightarrow \eta_Q V$ which are of substantial importance in discovering these new states. As is well known, the intermediate meson loop transitions have been an important nonperturbative transition mechanism in many processes, and their impact on the heavy quarkonium transitions has been noticed for a long time [31–33]. Recently, this mechanism has been applied to study B decays [34, 35] and the production and decays of exotic states [14–16, 27–30, 36–47], and a global agreement with experimental data was obtained. Inspired by this agreement, we shall adopt the effective Lagrangian approach (ELA) to study the decays $Z_{QV}^{(\prime)} \rightarrow \eta_Q V$.

The rest of this paper is organized as follows. In Sec. II, we present the possible molecular states composed of one pseudoscalar and one-vector (P - V) heavy mesons and two-vector (V - V) heavy mesons. In Sec. III, we will introduce the formulas for ELA. In Sec. IV, the numerical results are presented, and a brief summary is given in Sec. V.

II. HADRONIC MOLECULAR STATES

For simplicity, we use $Z_{QV}^{(\prime)}$ ($Q = c, b$ and $V = \rho, \omega, \phi$ and K^*) to represent the states with the mesonic constituents $H^{(*)}\bar{H}^{(*)}$, where the heavy quark is denoted as Q and $H^{(*)}\bar{H}^{(*)}$ represents the relevant heavy (anti)meson. The (I, I_3) are the same as V .

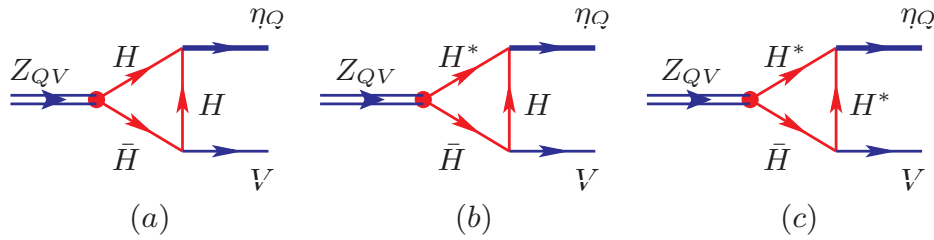
In Table. I, we collect the possible molecular states composed of one pseudoscalar and one vector

TABLE I: Possible molecular states Z_{QV} and Z'_{QV} with quantum numbers $J^{PC} = 1^{+-}$ for the neutral $Z_{QV}^{(\prime)}$.

| I, I_3 | Label | States | Label | States |
|-----------------------------|--------------------|--|----------------------------|---|
| 1, 1 | $Z_{c\rho}^+$ | $\frac{1}{\sqrt{2}}(\bar{D}^{*0}D^+ + \bar{D}^0D^{*+})$ | $Z_{c\rho}^{\prime+}$ | $\bar{D}^{*0}D^{*+}$ |
| 1, -1 | $Z_{c\rho}^-$ | $\frac{1}{\sqrt{2}}(D^{*0}D^- + D^0D^{*-})$ | $Z_{c\rho}^{\prime-}$ | $D^{*0}D^{*-}$ |
| 1, 0 | $Z_{c\rho}^0$ | $\frac{1}{2}[(\bar{D}^{*0}D^0 - D^{*-}D^+) + (\bar{D}^0D^{*0} - D^-D^{*+})]$ | $Z_{c\rho}^{\prime0}$ | $\frac{1}{\sqrt{2}}[(\bar{D}^{*0}D^{*0} - D^{*-}D^{*+})]$ |
| 0, 0 | $Z_{c\omega}^0$ | $\frac{1}{2}[(\bar{D}^{*0}D^0 + D^{*-}D^+) + (\bar{D}^0D^{*0} + D^-D^{*+})]$ | $Z_{c\omega}^{\prime0}$ | $\frac{1}{\sqrt{2}}[(\bar{D}^{*0}D^{*0} + D^{*-}D^{*+})]$ |
| 0, 0 | $Z_{c\phi}^0$ | $\frac{1}{\sqrt{2}}(D_s^{*+}D_s^- + D_s^+D_s^{*-})$ | $Z_{c\phi}^{\prime0}$ | $D_s^{*+}D_s^{*-}$ |
| $\frac{1}{2}, \frac{1}{2}$ | $Z_{cK^*}^+$ | $\frac{1}{\sqrt{2}}(\bar{D}^{*0}D_s^+ + \bar{D}^0D_s^{*+})$ | $Z_{cK^*}^{\prime+}$ | $\bar{D}^{*0}D_s^{*+}$ |
| $\frac{1}{2}, \frac{1}{2}$ | $Z_{cK^*}^-$ | $\frac{1}{\sqrt{2}}(D^{*0}D_s^- + D^0D_s^{*-})$ | $Z_{cK^*}^{\prime-}$ | $D^{*0}D_s^{*-}$ |
| $\frac{1}{2}, -\frac{1}{2}$ | $Z_{cK^*}^0$ | $\frac{1}{\sqrt{2}}(D^{*-}D_s^+ + D^-D_s^{*+})$ | $Z_{cK^*}^{\prime0}$ | $D^{*-}D_s^{*+}$ |
| $\frac{1}{2}, \frac{1}{2}$ | $Z_{c\bar{K}^*}^0$ | $\frac{1}{\sqrt{2}}(D^{*+}D_s^- + D^+D_s^{*-})$ | $Z_{c\bar{K}^*}^{\prime0}$ | $D^{*+}D_s^{*-}$ |
| I, I_3 | Label | States | Label | States |
| 1, 1 | $Z_{b\rho}^+$ | $\frac{1}{\sqrt{2}}(B^{*+}\bar{B}^0 + B^+\bar{B}^{*0})$ | $Z_{b\rho}^{\prime+}$ | $B^{*+}\bar{B}^{*0}$ |
| 1, -1 | $Z_{b\rho}^-$ | $\frac{1}{\sqrt{2}}(B^{*-}B^0 + B^-\bar{B}^{*0})$ | $Z_{b\rho}^{\prime-}$ | $B^{*-}B^{*0}$ |
| 1, 0 | $Z_{b\rho}^0$ | $\frac{1}{2}[(B^{*+}B^- - B^{*0}\bar{B}^0) + (B^+B^{*-} - B^0\bar{B}^{*0})]$ | $Z_{b\rho}^{\prime0}$ | $\frac{1}{\sqrt{2}}(B^{*+}B^{*-} - B^{*0}\bar{B}^{*0})$ |
| 0, 0 | $Z_{b\omega}$ | $\frac{1}{2}[(B^{*+}B^- + B^{*0}\bar{B}^0) + (B^+B^{*-} + B^0\bar{B}^{*0})]$ | $Z_{b\omega}^{\prime}$ | $\frac{1}{\sqrt{2}}(B^{*+}B^{*-} + B^{*0}\bar{B}^{*0})$ |
| 0, 0 | $Z_{b\phi}$ | $\frac{1}{\sqrt{2}}(B_s^{*+}\bar{B}_s + B_s^+\bar{B}_s^*)$ | $Z_{b\phi}^{\prime}$ | $B_s^{*+}\bar{B}_s^*$ |
| $\frac{1}{2}, \frac{1}{2}$ | $Z_{bK^*}^+$ | $\frac{1}{\sqrt{2}}(B^{*+}\bar{B}_s + B^+\bar{B}_s^*)$ | $Z_{bK^*}^{\prime+}$ | $B^{*+}\bar{B}_s^*$ |
| $\frac{1}{2}, -\frac{1}{2}$ | $Z_{bK^*}^-$ | $\frac{1}{\sqrt{2}}(\bar{B}^{*+}B_s + \bar{B}^+B_s^*)$ | $Z_{bK^*}^{\prime-}$ | $\bar{B}^{*+}B_s^*$ |
| $\frac{1}{2}, -\frac{1}{2}$ | $Z_{b\bar{K}^*}^0$ | $\frac{1}{\sqrt{2}}(B^{*0}\bar{B}_s + B^0\bar{B}_s^*)$ | $Z_{b\bar{K}^*}^{\prime0}$ | $B^{*0}\bar{B}_s^*$ |
| $\frac{1}{2}, \frac{1}{2}$ | $Z_{b\bar{K}^*}^0$ | $\frac{1}{\sqrt{2}}(\bar{B}^{*0}B_s + \bar{B}^0B_s^*)$ | $Z_{b\bar{K}^*}^{\prime0}$ | $\bar{B}^{*0}B_s^*$ |

(P - V) heavy mesons and two vector (V - V) heavy mesons. The quantum number is $J^{PC} = 1^{+-}$ for the neutral $Z_{QV}^{(\prime)}$.

III. DECAY AMPLITUDES

FIG. 1: The hadron-level diagrams for $Z_{QV} \rightarrow \eta_Q V$ via $H\bar{H}^* + c.c.$ intermediate heavy meson loops.

Our calculation is based on the assumption that the $Z_{QV}^{(\prime)}$ are S -wave $H^{(*)}\bar{H}^{(*)}$ molecular states.

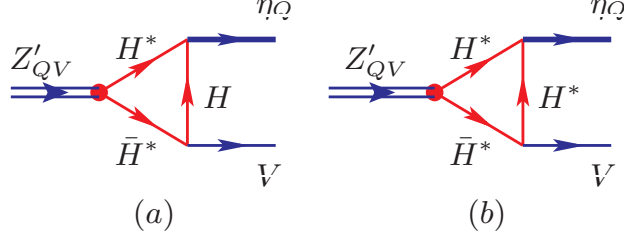


FIG. 2: The hadron-level diagrams for $Z'_{QV} \rightarrow \eta_Q V$ via $H^* \bar{H}^*$ intermediate heavy meson loops.

The relevant Lagrangians for $Z_{QV}^{(\prime)}$ coupled to a pair of heavy mesons can be expressed as

$$\mathcal{L}_{Z_{QV}, Z'_{QV}} = ig_{Z'_{QV}} \varepsilon^{\mu\nu\alpha\beta} \bar{H}_\alpha^{*\dagger} \partial_\mu Z'_{QV\nu} H_\beta^{*\dagger} + g_{Z_{QV}} (\bar{H}_\mu^{*\dagger} Z_{QV}^\mu H^\dagger + \bar{H}^\dagger Z_{QV}^\mu H_\mu^{*\dagger}) + H.c., \quad (1)$$

where H and H^* denote the pseudoscalar and vector heavy meson fields, respectively, i.e., $H^{(*)} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$ and $(B^{(*)+}, B^{(*)0}, B_s^{(*)0})$.

With the experimental measurements for $BR(Z_{b\rho}^+ \rightarrow B^+ \bar{B}^{*0} + \bar{B}^0 B^{*+}) = (86.0 \pm 3.6)\%$ and $BR(Z_{b\rho}'^+ \rightarrow B^{*+} \bar{B}^{*0}) = (73.4 \pm 7.0)\%$ [48], we obtain

$$g_{Z_{b\rho}} = 13.10_{-0.88}^{+0.83} \text{GeV}, \quad g_{Z_{b\rho}'} = 1.04_{-0.10}^{+0.10}. \quad (2)$$

If we assume that the total width of Z_c and Z_c' are saturated by the decay $Z_{c\rho}^+ \rightarrow D^+ \bar{D}^{*0} + \bar{D}^0 D^{*+}$ and $Z_{c\rho}'^+ \rightarrow D^{*+} \bar{D}^{*0}$, then the coupling constants are determined as follows,

$$g_{Z_{c\rho}} = 1.75_{-0.25}^{+0.24} \text{GeV}, \quad g_{Z_{c\rho}'} = 0.35_{-0.06}^{+0.05}. \quad (3)$$

The Lagrangian describing the interactions between S -wave heavy mesons and light vector mesons are as follows [34, 49]:

$$\begin{aligned} \mathcal{L} = & ig_{H^* H^* \nu} H_i^{*\nu\dagger} \overset{\leftrightarrow}{\partial}_\mu H_\nu^{*j} (\mathcal{V}^\mu)_j^i + 4if_{H^* H^* \nu} H_{i\mu}^{*\dagger} (\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu)_j^i H_\nu^{*j} \\ & - 2f_{H^* H \nu} \varepsilon_{\mu\nu\alpha\beta} (\partial^\mu \mathcal{V}^\nu)_j^i (H_i^{\dagger} \overset{\leftrightarrow}{\partial}^\alpha H^{*\beta j} - H_i^{*\dagger} \overset{\leftrightarrow}{\partial}^\alpha H^j) - ig_{HH\nu} H_i^{\dagger} \overset{\leftrightarrow}{\partial}_\mu H^j (\mathcal{V}^\mu)_j^i. \end{aligned} \quad (4)$$

In the chiral and heavy quark limits, the heavy meson couplings to the light vector meson have the following relationships [34, 49],

$$g_{HH\nu} = g_{H^* H^* \nu} = \frac{\beta g_V}{\sqrt{2}}, \quad f_{H^* H \nu} = \frac{f_{H^* H^* \nu}}{m_{H^*}} = \frac{\lambda g_V}{\sqrt{2}}, \quad (5)$$

where $f_\pi = 132$ MeV is the pion decay constant, the parameters g_V respect the relation $g_V = m_\rho/f_\pi$ [49]. We take $\beta = 0.9$, $\lambda = 0.56 \text{ GeV}^{-1}$, and $g = 0.59$ [50].

Based on the heavy quark symmetry [49, 51], the Lagrangians for the S -wave η_Q couplings to H and H^* are

$$\mathcal{L} = ig_{\eta_Q H^* H} H^{*\dagger\mu} (\partial_\mu \eta_Q H^\dagger - \eta_Q \partial_\mu H^\dagger) + g_{\eta_Q H^* H^*} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu H_\nu^{*\dagger} \partial_\alpha \eta_Q H_\beta^{*\dagger}. \quad (6)$$

TABLE II: A summary of meson masses (in units of MeV) adopted in the calculation.

| States | $Z_{b\rho}^+$ | $Z_{b\rho}'^+$ | η_b | B^* | B | B_s^* | B_s |
|--------|---------------|----------------|----------|---------|---------|---------|---------|
| Mass | 10607.2 | 10652.2 | 9391.0 | 5325.2 | 5279.25 | 5415.4 | 5366.77 |
| States | $Z_{c\rho}^+$ | $Z_{c\rho}'^+$ | η_c | D^* | D | D_s^* | D_s |
| Mass | 3891.5 | 4023.0 | 2981.0 | 2010.28 | 1869.62 | 2112.3 | 1968.49 |

The following couplings are adopted in the numerical calculations:

$$g_{\eta_Q H^* H} = 2g_Q \sqrt{m_{\eta_Q} m_{H^*} m_H}, \quad g_{\eta_Q H^* H^*} = 2g_Q \frac{m_{H^*}}{\sqrt{m_{\eta_Q}}}, \quad (7)$$

where $g_c = \sqrt{m_{J/\psi}}/(2m_D f_{J/\psi})$, with $f_{J/\psi} = 405 \pm 14$ MeV. $g_b = \sqrt{m_{\Upsilon(1S)}}/(2m_B f_{\Upsilon(1S)})$ and $f_{\Upsilon(1S)} = 715.2$ MeV is obtained via the experiment data [1]. Based on the relevant Lagrangians given above, the loop transition amplitudes for the transitions in Figs. 1 and 2 can be expressed in a general form in the effective Lagrangian approach as follows:

$$M_{fi} = \int \frac{d^4 q_2}{(2\pi)^4} \sum_{H^* \text{ pol.}} \frac{V_1 V_2 V_3}{a_1 a_2 a_3} \mathcal{F}(m_2, q_2^2) \quad (8)$$

where V_i ($i = 1, 2, 3$) are the vertex functions and $a_i = q_i^2 - m_i^2$ ($i = 1, 2, 3$) are the denominators of the intermediate heavy meson propagators. For example, in Fig. 1 (a), V_i ($i = 1, 2, 3$) are the vertex functions for the Z_{QV} , η_Q , and V mesons, respectively. V_i ($i = 1, 2, 3$) are the denominators for the intermediate H^* , \bar{H}^* , H , respectively. We adopt a monopole form factor,

$$\mathcal{F}(m_2, q_2^2) \equiv \frac{\Lambda^2 - m_2^2}{\Lambda^2 - q_2^2}, \quad (9)$$

where $\Lambda_i \equiv m_i + \alpha \Lambda_{\text{QCD}}$ and the QCD energy scale $\Lambda_{\text{QCD}} = 220$ MeV. This parameter scheme has been applied extensively in other works [29, 34, 37, 52, 53]. This form factor is supposed to offset the off-shell effects of the exchanged mesons [34, 54–56] and the form factor parameter should be determined by experimental information. The explicit expression of the transition amplitudes can be found in the Appendix V.

IV. NUMERICAL RESULTS

The molecule is the pole of the S matrix. As a result, it could be bound state (on a physical sheet below the threshold of constituent particles), virtual state (on an unphysical sheet below the threshold), or resonance (on an unphysical sheet above the threshold) [13, 57–61]. In our case, we assume these exotic states studied here are resonances and can decay into their constituent particles. In Table II, we list the meson masses involved in our calculation. Based on masses of

TABLE III: Predicted partial widths (in unit of MeV) of $Z_{cV} \rightarrow \eta_c V$. The parameter in the form factor is chosen as $\alpha = 2.0$ and $\alpha = 3.0$.

| | $\alpha = 2.0$ | | | | $\alpha = 3.0$ | | | |
|---|----------------|---------|--------|--------|----------------|---------|--------|--------|
| Binding energy | -20 MeV | -10 MeV | -5 MeV | -1 MeV | -20 MeV | -10 MeV | -5 MeV | -1 MeV |
| $Z_{c\rho}^+ \rightarrow \eta_c \rho^+$ | 0.44 | 0.42 | 0.41 | 0.41 | 0.76 | 0.73 | 0.71 | 0.70 |
| $Z_{c\rho}^0 \rightarrow \eta_c \rho^0$ | 0.44 | 0.42 | 0.41 | 0.37 | 0.76 | 0.73 | 0.72 | 0.64 |
| $Z_{c\omega}^0 \rightarrow \eta_c \omega$ | 0.43 | 0.41 | 0.40 | 0.35 | 0.75 | 0.72 | 0.70 | 0.63 |
| $Z_{c\phi}^0 \rightarrow \eta_c \phi$ | 0.09 | 0.08 | 0.08 | 0.08 | 0.15 | 0.15 | 0.14 | 0.14 |
| $Z_{cK^*}^+ \rightarrow \eta_c K^{*+}$ | 0.41 | 0.39 | 0.38 | 0.37 | 0.71 | 0.68 | 0.67 | 0.66 |
| $Z_{cK^*}^0 \rightarrow \eta_c K^{*0}$ | 0.40 | 0.39 | 0.38 | 0.35 | 0.71 | 0.68 | 0.67 | 0.63 |

TABLE IV: Predicted partial widths (in unit of keV) of $Z'_{cV} \rightarrow \eta_c V$. The parameter in the form factor is chosen as $\alpha = 2.0$ and $\alpha = 3.0$.

| | $\alpha = 2.0$ | | | | $\alpha = 3.0$ | | | |
|---|----------------|---------|--------|--------|----------------|---------|--------|--------|
| Binding energy | -20 MeV | -10 MeV | -5 MeV | -1 MeV | -20 MeV | -10 MeV | -5 MeV | -1 MeV |
| $Z_{c\rho}^{\prime+} \rightarrow \eta_c \rho^+$ | 12.43 | 11.26 | 10.69 | 10.25 | 22.88 | 20.74 | 19.72 | 18.91 |
| $Z_{c\rho}^{\prime0} \rightarrow \eta_c \rho^0$ | 12.51 | 11.32 | 10.73 | 9.37 | 23.01 | 20.84 | 19.77 | 17.45 |
| $Z_{c\omega}^{\prime0} \rightarrow \eta_c \omega$ | 11.81 | 10.67 | 10.09 | 8.81 | 21.76 | 19.66 | 18.62 | 16.44 |
| $Z_{c\phi}^{\prime0} \rightarrow \eta_c \phi$ | 1.95 | 1.74 | 1.63 | 1.55 | 3.65 | 3.26 | 3.07 | 2.93 |
| $Z_{cK^*}^{\prime+} \rightarrow \eta_c K^{*+}$ | 10.44 | 9.39 | 8.89 | 8.49 | 19.29 | 17.38 | 16.46 | 15.74 |
| $Z_{cK^*}^{\prime0} \rightarrow \eta_c K^{*0}$ | 10.30 | 9.26 | 8.76 | 8.37 | 19.05 | 17.16 | 16.24 | 15.53 |

the discovered states, we can get that the binding energy defined as $E_Z = m_{H^{(*)}} + m_{\bar{H}^{(*)}} - m_Z$ are about $(-12 \sim -2)$ MeV. These measurements have provided a range for the binding energy and we will choose a few illustrative values, $E_Z = (-20, -10, -5, -1)$ MeV, in the following. Choosing two values for the cutoff parameter α , we have predicted the partial decay widths, and the numerical results are collected in Tables III-VI.

As can be seen in Table III, the predicted partial widths of $Z_{cV} \rightarrow \eta_c V$ are less sensitive to the cutoff parameter α and the binding energy E_Z . The partial widths are about 1 MeV, except for $Z_{c\phi} \rightarrow \eta_c \phi$. It is noteworthy to recall that the experimental measurements for $\Gamma(Z_{c\rho})$ is 39.2 ± 10.5 MeV [8–10]. If these Z_{cV} have similar widths, our results would indicate a sizable branching fractions, at least 10^{-2} , for these decays. The partial width is only about 0.1 MeV for $Z_{c\phi} \rightarrow \eta_c \phi$, which is because the phase space is much smaller with the E_Z values considered in the calculations.

In Table IV, we list the partial widths of $Z'_{cV} \rightarrow \eta_c V$. The behavior is similar to that of $Z_{cV} \rightarrow \eta_c V$. The predicted partial widths are about several keV for $Z'_{c\phi} \rightarrow \eta_c \phi$ and tens of keV

TABLE V: Predicted partial widths (in units of MeV) of $Z_{bV} \rightarrow \eta_b V$. The parameter in the form factor is chosen as $\alpha = 2.0$ and $\alpha = 3.0$.

| | $\alpha = 2.0$ | | | | $\alpha = 3.0$ | | | |
|---|----------------|---------|--------|--------|----------------|---------|--------|--------|
| Binding energy | -20 MeV | -10 MeV | -5 MeV | -1 MeV | -20 MeV | -10 MeV | -5 MeV | -1 MeV |
| $Z_{b\rho}^+ \rightarrow \eta_b \rho^+$ | 1.97 | 1.93 | 1.91 | 1.89 | 3.22 | 3.16 | 3.13 | 3.11 |
| $Z_{b\rho}^0 \rightarrow \eta_b \rho^0$ | 1.97 | 1.93 | 1.91 | 1.89 | 3.22 | 3.16 | 3.13 | 3.11 |
| $Z_{b\omega}^0 \rightarrow \eta_b \omega$ | 1.94 | 1.91 | 1.89 | 1.87 | 3.19 | 3.13 | 3.11 | 3.08 |
| $Z_{b\phi}^0 \rightarrow \eta_b \phi$ | 0.43 | 0.42 | 0.42 | 0.41 | 0.74 | 0.73 | 0.72 | 0.72 |
| $Z_{bK^*}^+ \rightarrow \eta_b K^{*+}$ | 1.88 | 1.84 | 1.82 | 1.76 | 3.15 | 3.10 | 3.07 | 2.97 |
| $Z_{bK^*}^0 \rightarrow \eta_b K^{*0}$ | 1.87 | 1.83 | 1.81 | 1.75 | 3.14 | 3.08 | 3.05 | 2.95 |

TABLE VI: Predicted partial widths (in units of keV) of $Z'_{bV} \rightarrow \eta_b V$. The parameter in the form factor is chosen as $\alpha = 2.0$ and $\alpha = 3.0$.

| | $\alpha = 2.0$ | | | | $\alpha = 3.0$ | | | |
|--|----------------|---------|--------|--------|----------------|---------|--------|--------|
| Binding energy | -20 MeV | -10 MeV | -5 MeV | -1 MeV | -20 MeV | -10 MeV | -5 MeV | -1 MeV |
| $Z'_{b\rho}^+ \rightarrow \eta_c \rho^+$ | 105.01 | 99.46 | 96.73 | 94.58 | 196.68 | 186.11 | 180.93 | 176.84 |
| $Z'_{b\rho}^0 \rightarrow \eta_c \rho^0$ | 105.01 | 99.46 | 96.73 | 94.58 | 196.68 | 186.11 | 180.93 | 176.84 |
| $Z'_{b\omega}^0 \rightarrow \eta_b \omega$ | 101.52 | 96.06 | 93.39 | 91.27 | 190.10 | 179.71 | 174.63 | 170.61 |
| $Z'_{b\phi}^0 \rightarrow \eta_b \phi$ | 17.98 | 16.84 | 16.29 | 15.85 | 33.95 | 31.78 | 30.72 | 29.89 |
| $Z'_{bK^*}^+ \rightarrow \eta_b K^{*+}$ | 90.79 | 85.64 | 83.11 | 81.12 | 170.51 | 160.68 | 155.87 | 152.07 |
| $Z'_{bK^*}^0 \rightarrow \eta_b K^{*0}$ | 88.88 | 83.78 | 81.28 | 79.31 | 166.91 | 157.19 | 152.43 | 148.67 |

for other $Z'_{cV} \rightarrow \eta_c V$. If we assume that the Z'_{cV} have a width similar to $Z_c(4020)$, i.e., 9.7 ± 3.2 MeV, the corresponding branching ratios are about 10^{-3} , which is about 1-2 orders of magnitude smaller than that of $Z_{cV} \rightarrow \eta_c V$. As shown in Figs. 1 and 2, there are three kinds of diagrams for $Z_{cV} \rightarrow \eta_c V$, while there are only two kinds of diagrams for $Z'_{cV} \rightarrow \eta_c V$. Of course, there are still uncertainties coming from the coupling constants and off-shell effects arising from the exchanged particles of the loops, and the cutoff parameter can also be different in different decays. The numerical results would be lacking in accuracy and we expect to see experimental measurements on open charmed pair decays and the $\eta_c V$ decays in the near future.

The calculated partial widths of $Z_{bV} \rightarrow \eta_b V$ and $Z'_{bV} \rightarrow \eta_b V$ are presented in Tables V and VI, respectively. The behaviors are similar to $Z_{cV}^{(\prime)} \rightarrow \eta_c V$. The predicted widths of $Z_{bQ} \rightarrow \eta_b V$ are about a few MeV. The experimental measured total width of $Z_{b\rho}$ is (18.4 ± 2.4) MeV, taking into account the fact that these Z_{bV} should have similar total widths and that the corresponding branching ratios can reach up to 20%. The results for $Z_{b\phi}^0 \rightarrow \eta_b \phi$ are less than 1 MeV, which is

due to the suppressed phase space. For most $Z'_{bV} \rightarrow \eta_b V$, the partial widths are about 100-200 keV, which corresponds to branching ratios of 10^{-2} . Future experimental measurements can test our predictions and thus examine the properties of heavy quarkonium molecules.

V. SUMMARY

In this work, we have investigated the partners of the observed $Z_b(10610)/Z_b(10650)$ and $Z_c(3900)/Z_c(4025)$ states under the molecular framework. We have explored the isospin conserved decays $Z_{QV}/Z'_{QV} \rightarrow \eta_Q V$, respectively, via intermediate heavy meson loops. In this calculation, Z_{QV} and Z'_{QV} are assumed to be $H\bar{H}^* + c.c.$ and $H^*\bar{H}^*$ molecular states, respectively. The quantum numbers of the neutral partners of these two resonances are fixed to be $J^{PC} = 1^{+-}$.

For these decays, our results show that the α dependence of the partial widths are not less sensitive to some extent. The binding energy dependence is also quite stable. The predicted partial widths are of order of MeV for $Z_{QV} \rightarrow \eta_Q V$, which corresponds to the branching ratios of an order $10^{-2} \sim 10^{-1}$. For $Z'_{QV} \rightarrow \eta_Q V$, the partial widths are a few hundreds of keV and the branching ratios are about 10^{-3} . The results show some evidence of the universality of the molecular description of the observed $Z_c^{(\prime)}$ and $Z_b^{(\prime)}$, and they indicate that the intermediate heavy meson loops may be an important transition mechanism in the decays of the discovered exotic states, especially the initial states that are close to the two particle thresholds. We expect experiments to search for these $Z_{QV}^{(\prime)}$, the decays of $Z_{QV}^{(\prime)} \rightarrow \eta_Q V$, which will help us to test the universality of the molecular description of the $Z_{QV}^{(\prime)}$ and their transition mechanism deeply.

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Appendix A: The Transition Amplitude in ELA

In the following, we give the transition amplitudes for the intermediate heavy meson loops listed in Figs. 1 and 2 in the framework of the ELA. p_1 , p_2 , and p_3 are the four-vector momenta for initial states, final heavy quarkonium η_Q , and final light vector mesons V , respectively. q_1 , q_2 and q_3 are the four-vector momenta for the intermediate heavy mesons. ε_1 and ε_3 are the polarization vectors for initial state and final light vector mesons, respectively.

(i) $Z_{QV}^+ \rightarrow \eta_Q V$

$$\begin{aligned}
M_{HH^*[H^*]} &= \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_{QV}} \varepsilon_{1\mu}] [g_{\eta_Q H^* H} (p_2 + q_1)_\rho] [2g_{H^* H^* V} q_{2\theta} \varepsilon_3^{*\theta} g_{\phi\kappa} + 4f_{H^* H^* V} (p_3^\theta \varepsilon_{3\phi}^* - p_{3\phi} \varepsilon_3^{*\theta}) g_{\kappa\theta}] \\
&\quad \times \frac{i}{q_1^2 - m_1^2} \frac{i(-g^{\rho\phi} + q_2^\rho q_2^\phi/m_2^2)}{q_2^2 - m_2^2} \frac{i(-g^{\mu\kappa} + q_3^\mu q_3^\kappa/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2), \\
M_{H^* H[H^*]} &= \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_{QV}} \varepsilon_{1\mu}] [g_{\eta_Q H^* H} (p_2 - q_2)_\rho] [g_{HHV} (q_2 - q_3)_\lambda \varepsilon_3^{*\lambda}] \\
&\quad \times \frac{i(-g^{\mu\rho} + q_1^\mu q_1^\rho/m_1^2)}{q_1^2 - m_1^2} \frac{i}{q_2^2 - m_2^2} \frac{i}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2), \\
M_{H^* H^*[H]} &= \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_{QV}} \varepsilon_{1\mu}] [g_{\eta_Q H^* H^*} \varepsilon_{\rho\sigma\xi\tau} q_2^\rho p_2^\tau] [g_{H^* HV} \varepsilon_{\lambda\theta\phi\kappa} p_3^\lambda \varepsilon_3^{*\theta} (q_2 - q_3)^\phi] \\
&\quad \times \frac{i(-g^{\mu\xi} + q_1^\mu q_1^\xi/m_1^2)}{q_1^2 - m_1^2} \frac{i(-g^{\sigma\kappa} + q_2^\sigma q_2^\kappa/m_2^2)}{q_2^2 - m_2^2} \frac{i}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2). \tag{A1}
\end{aligned}$$

(ii) $Z_{QV}^+ \rightarrow \eta_Q V$

$$\begin{aligned}
M_{H^* H^*[H]} &= \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_{QV}^{(\prime)}} H^* H^* \varepsilon_{\mu\nu\alpha\beta} p_1^\mu \varepsilon_1^\nu] [g_{\eta_Q H^* H} (p_2 - q_2)_\lambda] [g_{H^* HV} \varepsilon_{\theta\xi\tau\kappa} p_3^\theta \varepsilon_3^{*\xi} (q_3 - q_2)^\tau] \\
&\quad \times \frac{i(-g^{\alpha\lambda} + q_1^\alpha q_1^\lambda/m_1^2)}{q_1^2 - m_1^2} \frac{i}{q_2^2 - m_2^2} \frac{i(-g^{\beta\kappa} + q_3^\beta q_3^\kappa/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2), \\
M_{H^* H^*[H^*]} &= \int \frac{d^4 q_2}{(2\pi)^4} [g_{Z_{QV}^{(\prime)}} H^* H^* \varepsilon_{\mu\nu\alpha\beta} p_1^\mu \varepsilon_1^\nu] [g_{\eta_Q H^* H^*} \varepsilon_{\lambda\sigma\xi\tau} q_2^\lambda p_2^\tau] \\
&\quad \times [2g_{H^* H^* V} q_{2\theta} \varepsilon_3^{*\theta} g_{\phi\kappa} + 4f_{H^* H^* V} (p_3^\theta \varepsilon_{3\phi}^* - p_{3\phi} \varepsilon_3^{*\theta}) g_{\kappa\theta}] \\
&\quad \times \frac{i(-g^{\alpha\xi} + q_1^\alpha q_1^\xi/m_1^2)}{q_1^2 - m_1^2} \frac{i(-g^{\phi\sigma} + q_2^\phi q_2^\sigma/m_2^2)}{q_2^2 - m_2^2} \frac{i(-g^{\beta\kappa} + q_3^\beta q_3^\kappa/m_3^2)}{q_3^2 - m_3^2} \mathcal{F}(m_2, q_2^2). \tag{A2}
\end{aligned}$$

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